# wQN $_*$ and wQN $^*$

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4. february 2010 Hejnice

(UPJŠ Košice)

wQN<sub>\*</sub> and wQN\*

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- quasi-normal convergence of sequence (f<sub>n</sub> : n ∈ ω) if there exists a limit function f and a sequence of positive reals (e<sub>n</sub> : n ∈ ω) converging to zero such that

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X has the property QN if each sequence of continuous functions converging to zero is converging to zero quasi-normally.

#### wQN-property

X has the property wQN if each sequence of continuous functions converging to zero has a subsequence converging to zero quasi-normally.

#### SSP-property

X has the property SSP if for each sequence of sequences  $\langle \langle f_{n,m} : m \in \omega \rangle : n \in \omega \rangle$  of continuous functions such that  $f_{n,m} \to 0$  for any  $n \in \omega$ , there exists a sequence  $\langle m_n : n \in \omega \rangle$  such that  $f_{n,m_n} \to 0$ .

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#### Some covers:

- $\gamma$  -cover  $\mathcal{U}$  every  $x \in X$  lies in all but finitely many members of  $\mathcal{U}$ 
  - a family of all countable open γ -covers: Γ(X), Γ
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 There is a model of ZFC, where all of these properties are equivalent.

#### Scheepers conjecture

Perfectly normal wQN-space has property  $S_1(\Gamma, \Gamma)$ .

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is **open** in a space *X*.

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Bukovsky L.: On  $wQN_*$  and  $wQN^*$  spaces (2008):

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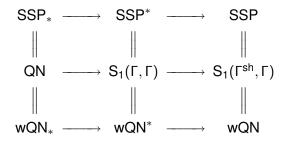
•  $wQN^* \rightarrow S_1(\Gamma, \Gamma)$ 

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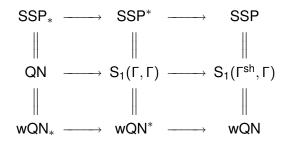
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(UPJŠ Košice)

wQN<sub>\*</sub> and wQN\*

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- $wQN_* \rightarrow wQN^*$
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(UPJŠ Košice)

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A space X has property (USC), if whenever  $\langle f_n : n \in \omega \rangle$  of **upper semicontinuous functions** with  $f(X) \subseteq [0, 1]$  converges to zero, there is  $\langle g_n : n \in \omega \rangle$  of **continuous functions** converging to zero such that

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• there is  $\langle f_{n,m} : m \in \omega \rangle$ , such that  $f_{n,m} \searrow f_n$  for every  $n \in \omega$ 

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•  $SSP_* \rightarrow (USC)$ 

Let  $\langle f_n : n \in \omega \rangle$  be a sequence of upper semicontinuous functions with  $f_n(X) \subseteq [0, 1]$  converging to zero. Then

- there is  $\langle f_{n,m} : m \in \omega \rangle$ , such that  $f_{n,m} \searrow f_n$  for every  $n \in \omega$
- $f_{n,m} f_n \searrow 0$  and  $f_{n,m} f_n$  is lower semicontinuous function
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#### • $SSP_* \rightarrow (USC)$

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•  $SSP_* \rightarrow (USC)$ 

- by (USC) there is ⟨⟨g<sub>n,m</sub> : m ∈ ω⟩ : n ∈ ω⟩ of continuous functions converging to zero such that f<sub>n,m</sub> ≤ g<sub>n,m</sub>
- by SSP<sub>\*</sub> there is  $\varphi \in {}^{\omega}\omega$  such that  $g_{n,\varphi(n)} \to 0$
- $f_{n,\varphi(n)} \rightarrow 0$
- $\bullet: \operatorname{SSP}_* \to \operatorname{SSP}^*$

 $\bullet: \mathsf{wQN}_* \to \mathsf{SSP}_*$ 

 $\bullet : w Q N_* \to w Q N^* \cap$ 

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- by (USC) there is ⟨⟨g<sub>n,m</sub> : m ∈ ω⟩ : n ∈ ω⟩ of continuous functions converging to zero such that f<sub>n,m</sub> ≤ g<sub>n,m</sub>
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•  $f_{n,\varphi(n)} \to 0$ 

•  $SSP_* \rightarrow SSP^*$ 

•  $wQN_* \rightarrow SSP_*$ 

•  $wQN_* \rightarrow wQN^*$ 

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- by SSP<sub>\*</sub> there is  $\varphi \in {}^{\omega}\omega$  such that  $g_{n,\varphi(n)} \to 0$

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•  $SSP_* \rightarrow SSP^*$ 

•  $wQN_* \rightarrow SSP_*$ 

•  $wQN_* \rightarrow wQN^*$ 

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- by SSP<sub>\*</sub> there is  $\varphi \in {}^{\omega}\omega$  such that  $g_{n,\varphi(n)} \to 0$

• 
$$f_{n,\varphi(n)} \rightarrow 0$$

#### • $SSP_* \rightarrow SSP^*$

•  $wQN_* \rightarrow SSP_*$ 

•  $wQN_* \rightarrow wQN^*$ 

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•  $f_{n,\varphi(n)} \rightarrow 0$ 

•  $SSP_* \rightarrow SSP^*$ 

• wQN $_* \rightarrow \mathsf{SSP}_*$ 

•  $wQN_* \rightarrow wQN^*$ 

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wQN<sub>\*</sub> and wQN<sup>\*</sup>

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- by (USC) there is ⟨⟨g<sub>n,m</sub> : m ∈ ω⟩ : n ∈ ω⟩ of continuous functions converging to zero such that f<sub>n,m</sub> ≤ g<sub>n,m</sub>
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• 
$$f_{n,\varphi(n)} \rightarrow 0$$

•  $SSP_* \rightarrow SSP^*$ 

#### • wQN $_* \rightarrow SSP_*$

•  $wQN_* \rightarrow wQN^*$ 

(UPJŠ Košice)

wQN<sub>\*</sub> and wQN<sup>\*</sup>

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$$f_{n,\varphi(n)} \to 0$$

•  $SSP_* \rightarrow SSP^*$ 

• wQN $_* \rightarrow SSP_*$ 

•  $wQN_* \rightarrow wQN^*$ 

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- $f^{-1}((r,1]) = (-f)^{-1}([-1,r))$
- $X \longrightarrow [-1,0]$  is upper semicontinuous function
- $\circ:\operatorname{SSP}^*([-1,0])\to\operatorname{SSP}_*([0,1])$

Properties related to semicontinuous functions depend on ranges of functions taken in their definitions.

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• 
$$f^{-1}((r,1]) = (-f)^{-1}([-1,r))$$

- $(-f): X \rightarrow [-1, 0]$  is upper semicontinuous function
- $SSP^*([-1,0]) \rightarrow SSP_*([0,1])$

Properties related to semicontinuous functions depend on ranges of functions taken in their definitions.

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Properties related to semicontinuous functions depend on ranges of functions taken in their definitions.

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• 
$$f^{-1}((r,1]) = (-f)^{-1}([-1,r))$$

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• 
$$SSP^*([-1,0]) \to SSP_*([0,1])$$

Properties related to semicontinuous functions depend on ranges of functions taken in their definitions.

**QN(A)**- as QN with functions restricted to range A **wQN(A)**, **wQN<sub>\*</sub>(A)**, **wQN<sup>\*</sup>(A)**- as wQN, wQN<sub>\*</sub>, wQN<sup>\*</sup> with functions restricted to range A **SSP(A)**, **SSP<sub>\*</sub>(A)**, **SSP<sup>\*</sup>(A)**- as SSP, SSP<sub>\*</sub>, SSP<sup>\*</sup> with functions restricted to range A

# $\mathsf{ON}([0, \infty]) = \mathsf{ON}([-1, 1]) = \mathsf{ON}([0, \infty]) = \mathsf{ON}([0, 1]) = \mathsf{ON}([0, 1$

 $\mathsf{MON}([0,1]) \equiv \mathsf{MON}([0,\infty]) \equiv \mathsf{MON}([0,\infty]) \equiv \mathsf{MON}([0,1]) \equiv \mathsf{MON}([0,1]$ 

 $\mathrm{SSP}(\mathbb{R}) \cong \mathrm{SSP}([-1,1]) \cong \mathrm{SSP}([0,\infty)) \cong \mathrm{SSP}([0,1]) \cong \mathrm{SSP}([0,\infty)) = \mathrm{SSP}([0,1]) \cong \mathrm{SSP}([0,\infty)) = \mathrm{SSP}([0,\infty)$ 

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# **QN(A)**- as QN with functions restricted to range A wQN(A), wQN<sub>\*</sub>(A), wQN<sup>\*</sup>(A)- as wQN, wQN<sub>\*</sub>, wQN<sup>\*</sup> with

restricted to range A

**SSP(A)**, **SSP<sub>\*</sub>(A)**, **SSP<sup>\*</sup>(A)**- as SSP, SSP<sub>\*</sub>, SSP<sup>\*</sup> with functions restricted to range A

# $\mathsf{ON}([0,\pi])=\mathsf{ON}([-1,\pi])=\mathsf{ON}([0,\infty))=\mathsf{ON}([0,\pi])=\mathsf{$

 $wQN([0] = wQN([-1,1]) = wQN([0,\infty)) = wQN([0,1]) = wQN([$ 

 $SSP(n) = SSP([-1, 1]) = SSP([0, \infty)) = SSP([0, 1]) = SSP($ 

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wQN<sub>\*</sub> and wQN\*

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#### QN(A)- as QN with functions restricted to range A

**wQN(A)**, **wQN<sub>\*</sub>(A)**, **wQN<sup>\*</sup>(A)**- as wQN, wQN<sub>\*</sub>, wQN<sup>\*</sup> with functions restricted to range A

**SSP(A)**, **SSP<sub>\*</sub>(A)**, **SSP<sup>\*</sup>(A)**- as SSP, SSP<sub>\*</sub>, SSP<sup>\*</sup> with functions restricted to range A

# $\mathsf{ON}([0,1])=\mathsf{ON}([0,\infty))=\mathsf{ON}([0,1])=\mathsf{ON}([0,\infty))=\mathsf{ON}([0,1])=\mathsf{O$

# $\mathsf{WQN}(\mathbb{R}) = \mathsf{WQN}([-1,1]) = \mathsf{WQN}([0,\infty)) = \mathsf{WQN}([0,1]) = \mathsf{WQN}([0,1]$

## $SSP(\mathbb{R}) = SSP([-1,1]) = SSP([0,\infty)) = SSP([0,1]) = SSP$

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## **QN(***A***)**- as QN with functions restricted to range *A* **wQN(***A***)**, **wQN**<sub>\*</sub>(*A*), **wQN**<sup>\*</sup>(*A*)- as wQN, wQN<sub>\*</sub>, wQN<sup>\*</sup> with functions restricted to range *A*

**SSP(A)**, **SSP<sub>\*</sub>(A)**, **SSP<sup>\*</sup>(A)** as SSP, SSP<sub>\*</sub>, SSP<sup>\*</sup> with functions restricted to range A

# $\mathsf{QN}(\mathbb{R}) \equiv \mathsf{QN}([-1,1]) \equiv \mathsf{QN}([0,\infty)) \equiv \mathsf{QN}([0,1]) \equiv \mathsf{QN}([0,1])$

# $\mathsf{WQN}([0,1]) = \mathsf{WQN}([-1,1]) = \mathsf{WQN}([0,\infty]) = \mathsf{WQN}([0,1]) = \mathsf{WQN}([0,1$

 $\mathrm{SSP}(\mathbb{R}) = \mathrm{SSP}([-1,1]) = \mathrm{SSP}([0,\infty)) = \mathrm{SSP}([0,1]) = \mathrm{SSP}([0,1]) = \mathrm{SSP}([0,\infty)) = \mathrm{SSP}([0,\infty)$ 

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QN(*A*)- as QN with functions restricted to range *A* wQN(*A*), wQN<sub>\*</sub>(*A*), wQN<sup>\*</sup>(*A*)- as wQN, wQN<sub>\*</sub>, wQN<sup>\*</sup> with functions restricted to range *A* SSP(*A*), SSP<sub>\*</sub>(*A*), SSP<sup>\*</sup>(*A*)- as SSP, SSP<sub>\*</sub>, SSP<sup>\*</sup> with functions restricted to range *A* 

 $QN(\mathbb{R}) \equiv QN([-1,1]) \equiv QN([0,\infty)) \equiv QN([0,1]) \equiv QN([0,1])$ 

 $wQN(\mathbb{R}) \equiv wQN([-1,1]) \equiv wQN([0,\infty)) \equiv wQN([0,1]) \equiv wQN([0,1])$ 

 $SSP(\mathbb{R}) \equiv SSP([-1,1]) \equiv SSP([0,\infty)) \equiv SSP([0,1]) \equiv SSP$ 

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**QN(***A***)**- as QN with functions restricted to range *A* **wQN(***A***)**, **wQN**<sub>\*</sub>(*A*), **wQN**<sup>\*</sup>(*A*)- as wQN, wQN<sub>\*</sub>, wQN<sup>\*</sup> with functions

restricted to range A

**SSP(A)**, **SSP<sub>\*</sub>(A)**, **SSP<sup>\*</sup>(A)**- as SSP, SSP<sub>\*</sub>, SSP<sup>\*</sup> with functions restricted to range A

$$\mathsf{QN}(\mathbb{R}) \equiv \mathsf{QN}([-1,1]) \equiv \mathsf{QN}([0,\infty)) \equiv \mathsf{QN}([0,1]) \equiv \mathsf{QN}$$

 $wQN(\mathbb{R}) \equiv wQN([-1,1]) \equiv wQN([0,\infty)) \equiv wQN([0,1]) \equiv wQN$ 

 $SSP(\mathbb{R}) \equiv SSP([-1,1]) \equiv SSP([0,\infty)) \equiv SSP([0,1]) \equiv SSP$ 

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**QN**(*A*)- as QN with functions restricted to range *A* **wQN**(*A*), **wQN**<sub>\*</sub>(*A*), **wQN**<sup>\*</sup>(*A*)- as wQN, wQN<sub>\*</sub>, wQN<sup>\*</sup> with functions

restricted to range A

**SSP(A)**, **SSP<sub>\*</sub>(A)**, **SSP<sup>\*</sup>(A)**- as SSP, SSP<sub>\*</sub>, SSP<sup>\*</sup> with functions restricted to range A

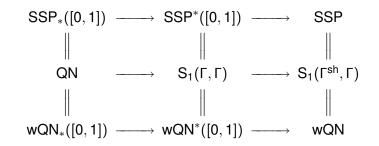
$$\mathsf{QN}(\mathbb{R}) \equiv \mathsf{QN}([-1,1]) \equiv \mathsf{QN}([0,\infty)) \equiv \mathsf{QN}([0,1]) \equiv \mathsf{QN}$$

 $wQN(\mathbb{R})\equiv wQN([-1,1])\equiv wQN([0,\infty))\equiv wQN([0,1])\equiv wQN$ 

 $SSP(\mathbb{R})\equiv SSP([-1,1])\equiv SSP([0,\infty))\equiv SSP([0,1])\equiv \textbf{SSP}$ 

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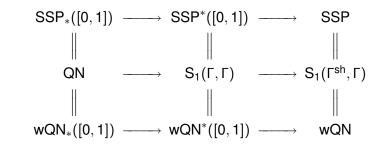
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What are the relations among  $wQN_*([-1,1])$ ,  $SSP_*([-1,1])$ , ...?

UPJŠ Košice)	wQN <sub>*</sub> and wQN <sup>*</sup>	4. february 2010 Hejnice	14 / 16

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(UPJŠ Košice)	wQN <sub>*</sub> and wQN <sup>*</sup>	4. february 2010 Hejnice	14 / 16

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# Equivalent properties

 $wQN^{*}([0, 1])$  $wQN^{*}([0, \infty))$ 

 $SSP_{*}([0, 1])$   $SSP_{*}([0, \infty))$   $SSP_{*}([-1, 1])$   $SSP^{*}([-1, 1])$   $SSP_{*}(\mathbb{R})$  $SSP^{*}(\mathbb{R})$ 

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wQN<sub>\*</sub> and wQN\*

 $wQN^{*}([0, 1])$  $wQN^*([0,\infty))$ 

 $SSP^{*}([0, 1])$  $SSP^*([0,\infty))$ 

wQN<sub>\*</sub> and wQN<sup>\*</sup>

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$$wQN^{*}([0, 1]) \\ wQN^{*}([0, \infty))$$

$$SSP^{*}([0, 1])$$
  
 $SSP^{*}([0, \infty))$ 

$$\begin{split} & wQN_*([0,1]) & SSP_*([0,1]) \\ & wQN_*([0,\infty)) & SSP_*([0,\infty)) \\ & wQN_*([-1,1]) & SSP_*([-1,1]) \\ & wQN^*([-1,1]) & SSP^*([-1,1]) \\ & wQN_*(\mathbb{R}) & SSP_*(\mathbb{R}) \\ & wQN^*(\mathbb{R}) & SSP^*(\mathbb{R}) \end{split}$$

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# Thanks for your attention!

(UPJŠ Košice)

wQN<sub>\*</sub> and wQN<sup>\*</sup>

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